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Application of shannon entropy in the calculation of the artistic style of xin'an painting school

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Abstract. Artists and critics generally make artistic analysis of paintings based on their experience and subjective consciousness. In order to calculate the art style of the Xin'an School of Painting objectively, this paper attempts to propose several digital features including redundancy, degree of order and complexity to describe the artistic style of paintings algorithm under the framework of computable aesthetics on the basis of Shannon entropy in information theory. Taking the five painters of different periods as the research object, the digital features of the selected works are calculated and analyzed. The experimental results show that the proposed digital features can reflect the artistic style and the differences between different artists of the Xin'an School of Painting in different periods to some extent, which are basically consistent with the subjective aesthetic perception of the School.

Key words. art style, Shannon entropy, redundancy, degree of order, complexity.

1. Introduction

As an objective digital processing tool, computer can figure out some characteristics of the style of painting artworks, playing an important role in the authenticity and the identification of the times of the painting. As a result, it is in an increasing demands to use computers to assist people to analyze paintings objectively.

At present, there are mainly two directions to do research into painting with computer-aided technology: the evaluation of aesthetic quality and the understanding of artistic style. Sparse coding and information theory are adopted in the understanding of art styles. Sparse coding algorithms began to be used for annotation of images (Liu et al., 2014), and later the algorithms was improved from the perspective of perceptions (Amiri et al., 2014). The image can be recovered by real-time sparse

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coding when the sparse structure meets with the Gaussian mixture (Dong et al., 2015). Taking advantage of sparse coding to create a stroke dictionary of Bruegel's works, the authenticity of paintings can be identified by comparing kurtosis values (Hughes et al., 2010). It is necessary to simulate the visual image by digital features to understand the image as a whole with similar semantics and visual similarity. The color and texture features of the image can be described by combining the local inverse color space extrema model and the current local extremal model (Reddy et al., 2015). Image invariant invariance as the global content descriptor of the image also provides an overall understanding of the image (Yang et al., 2015). The objective image quality evaluation is to obtain the quantitative index in accordance with the experimental model to evaluate the image quality by simulating the human visual system, and extracts one or more characteristic values of the image with a specific mathematical formula to calculate the image quality, so as to obtain consistent results with the subjective image quality. For example, methods for evaluating specific distortions (Golestaneh et al., 2014; Corchs et al., 2014), and local spatial entropy with image and spectral entropy to construct a two-level image quality evaluation model (Liu et al., 2014).

2. Theory of Art Style Calculation

Information theory is a science that uses mathematical statistics to study the measurement, transmission and transformation rule. In image processing, if the value of pixels in image is as a discrete random variable, the information theory can be used to quantify and analyze the painting works. Here are some concepts in information theory.

2.1. Shannon Entropy

Let X be a discrete random variable whose possible values is $x_1, x_2, \cdot, x_k$, so the sample space can be expressed as $\{x_1, x_2, \cdot, x_k\}$. Taking any element x_i in the sample space as X, and then the probability can be expressed as $p(x_i) = \{X = x_i\}$, $x_i \in X$.

Shannon entropy H (X) of the random variable X is defined as:

$$\mathrm{H}(\mathrm{X}) = -\sum_{\mathrm{x_i} \in \mathrm{X}} \mathrm{p}(\mathrm{x_i})\,\mathrm{lbp}(\mathrm{x_i}) \tag{1}$$

It refers to the uncertainty measure of X, and satisfies $0 \le \mathrm{H}(\mathrm{X}) \le \mathrm{lb}|\mathrm{X}|$.

2.2. Conditional Entropy

If an information channel X→Y is created between two random variables X and Y (X is input and Y is output), the joint probability density function is

$\mathrm{p}(\mathrm{x_i}, \mathrm{y_j}) = \mathrm{P}(\mathrm{X} = \mathrm{x_i}, \mathrm{Y} = \mathrm{y_j})$, and the conditional probability density function is

$p(x_i|y_j) = P(X = x_i|Y = y_j)$. Conditional entropy $H(X|Y)$ can be defined as:

$$H(X|Y) = -\sum_{x_i \in X, y_j \in Y} p(x_i, y_j)\, \mathrm{lb}\, p(x_i|y_j) \tag{2}$$

It refers to the average uncertainty of X is obtained given the output value of Y.

2.3. Mutual Information

Mutual information represents the amount of information when one thing occurs and another one can be deduced from the former one. The mutual information $I(X, Y)$ between the random variables X and Y is defined as:

$$I(X, Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) \tag{3}$$

It refers to the amount of reduction in the uncertainty of X given the Y information, that is to say the same amount of information between X and Y.

3. Art Style Calculation

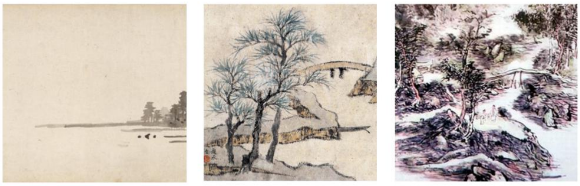
For a color image I with N pixels, if the color is stored in a manner of 24 bit, each pixel corresponds to a color value in the RGB color space. If the color value of each pixel is regarded as a random variable X_{RGB}, then the random variable X_{RGB} sample space contains 256^3color values. In this paper, the image brightness space X_L is also adopted, which corresponds to the brightness information of the image and satisfies $X_L = [0, 255]$. In Matlab, the probability distribution of random variables X_{RGB} and X_L can be obtained by using histogram statistics method. The maximum Shannon entropy H_{max} of these two random variables is 24 and 8, respectively.

3.1. Palette Redundancy

Paintings are composed of a variety of colors, each of which has uncertainties in its color palette. The uncertainty can be expressed in terms of Shannon entropy $H(X_{RGB})$ of the palette. If an image is used in a relatively small number of colors, its uncertainty is less and the extra information is more. That is, the palette has smaller Shannon entropy with higher redundancy. When painting, painters determine the color of the painting according to the needs of the work or their personal habits, and then the uncertainty of the color palette will be reduced. The creative process can be seen as the reduction of the Shannon entropy of the palette, the reduction of the palette uncertainty is expressed by the color palette redundancy M_B, the formula is:

$$M_B = \frac{H_{max} - H(X_{RGB})}{H_{max}} \tag{4}$$

Where H_{max} is the maximal entropy, that is, the color values of all the pixels are different, and the color palette has the maximum uncertainty.



(a)Li Liufang (b)Zha Shibiao (c)Huang Binhong

Fig. 1. Xin'an paintings

The three Xin'an paintings shown in Figure 1 are part of the representative works by Wang Zhirui, Zha Shibiao and Huang Binhong. The M_B values of three works are calculated in Matlab according to formula (4).The H and M_B values is 7.2851 and 0.5916 in Figure 1(a),11.0772 and 0.4869 in Figure 1(b),13.7016 and 0.3975 in Figure 1(c).

Paintings can be understood as a painter uses the brush to draw a chaotic map according to a certain law on the canvas. The degree of order of the images can be used to characterize the degree of the artist's creation. The order of the images includes Kolmogorov ordering and Shannon entropy ordering.

3.2. Kolmogorov Order Degree

The Kolmogorov order degree can be calculated from the differences between all the original information of the image and the information retained after compression. All the original information of the image can be represented by the maximal Shannon entropy H_{max} of the image multiplied by the number of its pixels N. When the computer removes the data redundancy of one image, the shortest data that can be obtained by the computer is the image compression. The retention information is the image's Kolmogorov complexity $K(I)$. The formula is:

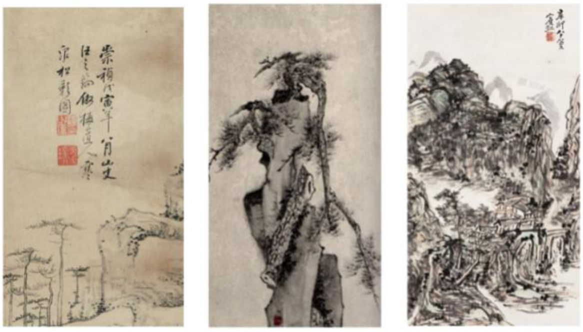
$$M_K = \frac{N \times H_{max} - K(I)}{N \times H_{max}} \tag{5}$$

During processing, the value of $K(I)$ can be approximately regarded as the compression space for image compression. The author of this paper uses JPEG compression with high-efficiency and high-quality to help calculate Kolmogorov order degree M_K.

The M_K values and compression ratios is 0.8944 and 6.1218 in Figure 2(a),0.8366 and 5.3875 in Figure 2(b),0.8107 and 5.2829 in Figure 2(c),which are programmed and calculated in Matlab according to formula (5).

3.3. Shannon Entropy Degree of Order

The degree of order of the metric images is not limited to the areas before and after the image compression. Given an image palette Shannon entropy $H(X_{RGB})$, the range of color types can first be determined, and then the degree of its image order. At that moment, the order can be called Shannon entropy degree of order, and its value can be obtained by replacing the maximal Shannon entropy H_{max} in

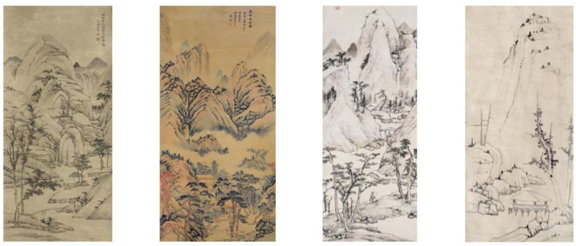


(a)Wang Zhirui (b)Jian Jiang (c)Huang Binhong

Fig. 2. Xin'an paintings

Kolmogorov order with the known palette Shannon entropy $\mathrm{H}(\mathrm{X}_{\mathrm{RGB}})$. The formula is:

$$M_H = \frac{N \times H(X_{RGB}) - K(I)}{N \times H(X_{RGB})} \tag{6}$$



(a) Jian Jiang (b)Zha Shibiao (c) Zha Shibiao (d) Wang Zhirui

Fig. 3. The figures of M_K and M_H in different paintings

The M_{K} and M_H values is 0.7450 and 0.6097 in Figure 3(a),0.7451 and 0.6312 in Figure 3(b),0.6969 and 0.5626 in Figure 3(c),0.7052 and 0.5629 in Figure 3(d),which are programmed and calculated in Matlab according to formula (6).

3.4. The Complexity of the Work

Areas with the same or similar color values in a painting are called homogeneous areas, while with large differences are heterogeneous areas, so an image can be divided into n blocks. Define n as the complexity of the work.

The mutual information $I(C,R)$ can represent the correlation between the color C and the area R in the image, where C represents the color distribution and R represents the distribution of the different color areas. The formula can be calculated according to equation (3):

$$I(C,R) = \sum_{\mathrm{c} \in C} p(c) \sum_{\mathrm{r} \in R} p(r|c)\, \mathrm{lb} \frac{p(r|c)}{p(r)} \tag{7}$$

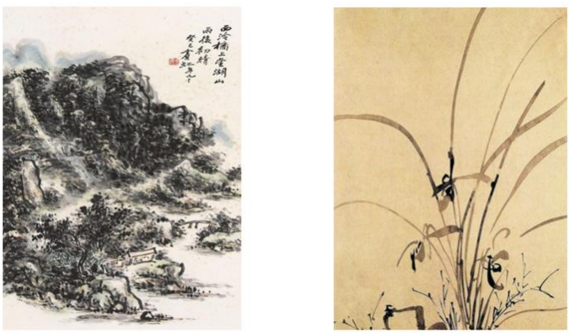
The ratio MI between mutual information $I(C,R)$ and the Shannon entropy $H(C)$ of the whole image is taken as the threshold for judging whether it is homogeneous or not. When the MI value between an area and the adjacent pixel is less than the given threshold, it is considered that this area with the pixel is a homogeneous area; otherwise it is a heterogeneous area.

In dividing, the whole image is taken as a large area R firstly, the images are scanned and divided by greedy algorithm in the order from left to right and top to bottom, the maximal MI of two different dividing directions are calculated by choosing the maximum value from it to determine the appropriate direction and the best point of division. Furthermore, the two regions after division can be judged. If the value is smaller than the set threshold, it is considered that the two image blocks obtained by the division are the homogeneous regions, and the division is stopped. Otherwise, the above process recursion is implemented. Since the final segmentation of the image is obtained by MI, the final blocks number n can be found by its inverse function. The formula is:

$$n = MI^{-1}\left(\frac{I(C,R)}{H(C)}\right) \tag{8}$$

This paper uses the brightness space X_L to calculate $I(C,R)$ and $H(C)$ to reduce the calculation of mutual information.

According to the formula (8) , the values of the two works in Figure 4 is 528 and 147 when the threshold is 0.15.



(a)Huang Binhong (b)Li Liufang

Fig. 4. The values of n in different paintings

4. Analysis of the Art Style of Xin'an School of Painting

Experimental environment: CPU 2.4 GHz, memory 8 GB. In the Matlab 2014b environment, the programming has realized the calculation of redundancy, degree of order and complexity. The paintings are from the representative of the Xin'an School of painting at different periods: Li Liufang, Jian Jiang, Zha Shibiao, Wang Zhirui, and Huang Binhong. 20 works are selected from each artist and the average character value of each work can be found, as shown in Table 1.

Table 1.The mean values of feature values from twenty paintings of every painter

Painter	H	M_B	M_K	M_H	n
Huang Binhong	14.4617	0.3641	0.8108	0.6293	628
Zha Shibiao	12.8519	0.4648	0.8897	0.6320	342
Li Liufang	12.7207	0.4858	0.8402	0.6374	265
Wang Zhirui	12.1016	0.4923	0.8919	0.6968	337
Jian Jiang	11.9650	0.5019	0.8394	0.7005	320

4.1. Analysis of the Painter's Color Habit

Xin'an School painters use ink as the main raw material with adding different proportions of water to draw different shades of black, and use ocher, azurite, cyanine, etc. to paint rocks, leaves and so on. The color of the painting is single, mainly in black, white and gray. In the works, there are few kinds of colors, and the uncertainty of color in average is relatively low with low H values and high M_B values, as shown in Table 1.

In Figure 1(a), except for a few black blocks, the rest consists of large areas of white space and very light black blocks, so the uncertainty of the palette is very low, whereas the palette redundancy will be high; Figure 1 (b) contains more cyan and brown areas, so its H value is also higher than in Figure 1 (a); correspondingly, M_B is lower than in Figure 1(a). However, the lowest value M_B and the highest value H appear in Figure 1(c), because Huang Binhong's works are rich in yellow, gray and blue gray and other color areas.

4.2. Analysis of the Degree of Order of Paintings

In the process of painting, the Xin'an painters fully embody the "freehand" features in Chinese painting art. They do not emphasize the background, and their paintings have white space except the explicit theme. Therefore, their works have a very significant regularity, indicating that their ordinal values M_K and M_H are high, as shown in Table 1.

Both the M_K value and the compression ratio of the Figure 2(a) are higher than those of the Figure 2(c). That is because Wang Zhirui likes to use heavy black lines to paint outlines of large rocks, which form a very simple picture structure, showing strong regularity with well degree of order and compression ratio. While Huang Binhong pays great attention to the details of pines and rocks during his paintings, he sketches out their rich forms with various lines. Therefore, in his works, the degree of order and compression ratio is lower than works of Wang Zhirui and Jian Jiang. Jian's works are full of lines to deal with mountains and trees into geometric shape like more mountains and less trees to the local and the general painting, so the degree of order and compression ratio are between the above two painters.

When the Kolmogorov order of some painting works is consistent, it can be measured by Shannon entropy order. As shown in Figure3 (a) and (b), the $M_K$value Kolmogorov order of degree is almost the same, but the two works are from different artists, so $H(X_{RGB})$can be used to judge the degree of order. The M_H value of Figure3 (a) and (b) shows some differences. Similarly, the regularities of color distributions on canvas in Figure 3 (c) and (d) are similar, so the order of Shannon entropy M_H is almost the same, the value of Kolmogorov order degree is different.

4.3. Analysis of the Complexity of Painting

The work of Xin'an School of Painting uses lines to make an outline, relying on the permeability of rice paper to seep out the ink, and the color of the painting emphasizes the inherent color of the object rather than the conditional color in special light. Therefore, the complexity of the Xin'an painting is generally similar. However, Li Liufang and Huang Binhong, also represented by the Xin'an School of Painting, have different levels of complexity in their works because of their different styles of painting.

Figure 4 shows two works drawn by Huang Binhong and Li Liufang. From the images, it can be seen that the blank areas of the background, such as the pine trees, rocks and hut, and the blank area of the background and the black parts, in which the bamboo leaves and the bamboo poles are concentrated in Figure (b), all of them can be distinguished from each other, and the part with lots of different colors is finely divided. From Figure 4 and Table 1, the value of n in Huang's work is higher than that of Li's by taking the same threshold. Because the objects are rocks and trees in Li's works, and the local part is drawn by black or gray after the outline is done, so there are more homogeneous area. While in the works of Huang, in addition to different levels of black, azurite and flower blue are commonly used in painting mountains and trees to stress the detail, so there are more heterogeneous areas.

5. Conclusion

Based on Shannon entropy and mutual information in information theory, this paper interprets the color value of pixel as the message in information theory and defines three digital features of redundancy, order of degree and complexity. Five representative painters from Xin'an School of painting are chosen as the research object to calculate the average value of the digital characteristics of their paintings. Through the analysis of the eigenvalues of selected works, it is found that the commons of artistic style and the differences among Xin'an painters in the application of ink color, the order degree of compositions and the complexity of works, are basically in line with the evaluations of critics in the history of painting.

Some digital features can be added to further explore the art style of painting, and digital features can also be applied to non-photo realistic rendering, style transfer and other aspects.

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